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Seasonal Nonlinear Long Memory Model for the US Inflation Rates

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Abstract This paper studies whether to describe nonlinearity, seasonality and long memory simultaneously in US inflation rates. To this aim, we define a seasonal FISTAR (SEA-FISTAR) model as an extension of FISTAR model proposed by Van Dijk et al. (J Economet 102:135–165, 2002). The results show that when combining these three features, the description of the inflation is improved and that seasonality changes smoothly with the regimes.

Keywords Long memory · Seasonality · Smooth transition autoregression

JEL Classification C22 · C51 · E31

1 Introduction

Long memory and nonlinearity have a large history in times series analysis for macroeconomic data. Long memory models are useful in economics as a parsimonious way of modelling highly persistent process. On other hand, nonlinear time series can be extremely informative about some aspects of the economy.

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In literature, long memory and nonlinearity are two principal features for US inflation rates. Long memory can be described by a fractionally integrated model (FI) introduced by Granger and Joyeux (1980) and Hosking (1981). Many studies provide strong evidence of long memory in US inflation rates, see Hassler and Wolters (1995); Bos et al. (1999) and many other papers cited in Baillie (1996). Nonlinearity feature of US inflation is well documented in the literature. For instance, Garcia and Perron (1996) have used Markov switching autoregressive models to describe US interest rate and inflation rates. In the same way, Evans and Lewis (1995) estimated a Markov switching model with two regimes for US inflation data. Ben Aïssa et al. (2004) used Bai and Perron's and spectral density methods for detecting structural changes in US inflation data. Warne and Anders (2006) studied two states Markov switching VAR model for unemployment and inflation. Ghysels and Osborn (2001) used periodic approaches of seasonal time series. Arteche (2002) studied seasonal fractional integration models for inflation. Franses and Ooms (1997) used periodic long memory models for inflation.

Generally, nonlinear dynamics were a relevant issue in the business cycles. In most cases, empirical studies on business cycles have been done with seasonally adjusted data. This implies that seasonal cycles are both regular and devoid of any economic information. But many works suggest that seasonal fluctuations and business cycles are closely related (see Miron and Beaulieu 1996; Franses 1996; Franses and Paap 1999). In contrast, Van Dijk et al. (2003) found that seasonality in quarterly industrial production for the G7 countries changes over time. They conclude that this change is mainly due to gradual institutional and technological change.

In view of this, we propose an univariate time series model able to capture nonlinearity, long memory and seasonality fluctuations. Up to now, there are no works to bring together these features. Franses et al. (2000) proposed a seasonal smooth transition autoregression to capture nonlinearity and seasonal fluctuations in US unemployment rate. Later, van Dijk et al. (2002) proposed a fractional integrated smooth transition autoregressive model (FISTAR) to combine the concepts of fractional integration and smooth transition nonlinearity for US unemployment rate. In our paper, we combine these two works to have a new model able to describe three features which are fractional integration, nonlinearity (see Teräsvirta 1994, 1998; Granger and Teräsvirta 1993) and seasonal fluctuations for a possible change according to regimes (see Canova and Ghysels 1994).

The paper is organized as follows. In Sect. 2, we introduce the seasonal fractional integrated smooth transition autoregressive model (SEA-FISTAR). In Sect. 3 we give the empirical specification procedures for our SEA-FISTAR model based on the method proposed in Teräsvirta (1994) for the basic STAR model. The empirical specification consists of some steps as nonlinearity test, estimation and misspecification tests. In Sect. 4 the model is fitted to quarterly US inflation rate. In addition, the SEA-FISTAR model with its particular cases is compared with a seasonal fractional integrated linear autoregressive model (SEA-ARFI). Finally, Sect. 5 concludes.

2 Models

The seasonal FISTAR model is an extension of FISTAR model introduced by van Dijk et al. (2002). Our modification consists in introducing an explicit description of the seasonal pattern of the series, i.e. using seasonal dummy variables (See Franses et al. 2000). The seasonal FISTAR model is able to describe seasonality, non-linearity and long memory in the time series.

The seasonal FISTAR model is given by:

$$(1-L)^d \, y_t = x_t \tag{2.1}$$

with

$$\begin{aligned} x_t = &\left(\sum_{s=1}^{S} D_{s,t}\mu_{1,s}\right)\left(1 - F\left(S_{F,t};\gamma_F,c_F\right)\right) + \left(\sum_{s=1}^{S} D_{s,t}\mu_{2,s}\right) \times F\left(S_{F,t};\gamma_F,c_F\right) \\ &+\left(\sum_{i=1}^{p}\phi_{1,i}x_{t-i}\right)\left(1 - G\left(S_{G,t};\gamma_G,c_G\right)\right) + \left(\sum_{i=1}^{p}\phi_{2,i}x_{t-i}\right) \\ &\times G\left(S_{G,t};\gamma_G,c_G\right) + \varepsilon_t \end{aligned} \tag{2.2}$$

d is the fractional integration degree of the process (see Granger and Joyeux 1980). L is the backshift operator such that $Ly_t = y_{t-1}$. $D_{s,t}$ are the seasonal dummy variables with $D_{s,t} = 1$ in season s, 0 elsewhere. S is the number of season in one period, $\mu_{j,s}$ are the seasonal means in regime j, $j = 1, 2$. $\phi_{j,i}$, $i = 1, \ldots, p$, are the autoregressive parameters in regime j. $\varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$. $F\left(S_{F,t};\gamma_F,c_F\right)$ and $G\left(S_{G,t};\gamma_G,c_G\right)$ are two logistic transition functions which are defined as (see van Dijk et al. 2002):

$$F\left(S_{F,t};\gamma_F,c_F\right) = \left[1+\exp\left(-\frac{\gamma_F}{\sigma_{S_{F,t}}}\left(S_{F,t}-c_F\right)\right)\right]^{-1} \tag{2.3}$$

and

$$G\left(S_{G,t};\gamma_G,c_G\right) = \left[1+\exp\left(-\frac{\gamma_G}{\sigma_{S_{G,t}}}\left(S_{G,t}-c_G\right)\right)\right]^{-1} \tag{2.4}$$

$S_{F,t}$ and $S_{G,t}$ are the two transition variables of the functions F and G respectively. They can be a function of $(y_{t-1}, y_{t-2}, \ldots)$ or a linear deterministic trend. γ_F and γ_G are the two parameters whose determine the smoothness of the functions F and G respectively. $\sigma_{S_{F,t}}$ and $\sigma_{S_{G,t}}$ are the standard deviations of the two transition variables $S_{F,t}$ and $S_{G,t}$ respectively. Finally, c_F and c_G are the thresholds of the transition from one regime to another. F and G change smoothly from zero to one as the corresponding transition variables $S_{F,t}$ and $S_{G,t}$ increase.

The transition functions F and G are assumed to be ranging from 0 to 1. The extremes of $G = 0$ and $G = 1$ may be associated with two different regimes that is the lower regime and the upper regime respectively. The speed of transition of G is determined by γ_G. If γ_G is small, then the transition of G from 0 to 1 takes a long period of time. However, if γ_G is large, the transition function G moves from 0 to 1 very quickly and when γ_G is near $+\infty$, this function change value from 0 to 1 instantaneously. F has the same properties of G according to the value of γ_F.

Equations 2.1–2.2 are the general representation of the seasonal FISTAR models. However, some particular cases can be discussed according to the variables and the parameters values of the transition functions.

When $\gamma_F = \gamma_G = 0$, (2.2) reduces to a linear seasonal autoregressive model, that is,

$$x_t = \sum_{s=1}^{S} D_{s,t}\mu_s + \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t. \tag{2.5}$$

If $\gamma_F = 0$, then $F\left(S_{F,t}; \gamma_F, c_F\right) = \frac{1}{2}$ for all values of $S_{F,t}$ and (2.2) becomes:

$$\begin{aligned} x_t &= \sum_{s=1}^{S} D_{s,t}\mu_s + \left(\sum_{i=1}^{p} \phi_{1,i} x_{t-i}\right)(1 - G(S_t; \gamma, c)) \\ &\quad + \left(\sum_{i=1}^{p} \phi_{2,i} x_{t-i}\right) \times G(S_t; \gamma, c) + \varepsilon_t \end{aligned} \tag{2.6}$$

in this case only the autoregressive parameters change between regimes. There is no regime switching in the seasonal component.

When $F\left(S_{F,t}; \gamma_F, c_F\right) = G\left(S_{G,t}; \gamma_G, c_G\right)$, the seasonal FISTAR model can take the following representation:

$$\begin{aligned} x_t &= \left(\sum_{s=1}^{S} D_{s,t}\mu_{1,s} + \sum_{i=1}^{p} \phi_{1,i} x_{t-i}\right) \times (1 - G(S_t; \gamma, c)) \\ &\quad + \left(\sum_{s=1}^{S} D_{s,t}\mu_{2,s} + \sum_{i=1}^{p} \phi_{2,i} x_{t-i}\right) G(S_t; \gamma, c) + \varepsilon_t \end{aligned} \tag{2.7}$$

In this case, the seasonal FISTAR model is restricted in a way that both seasonal and autoregressive parameters change simultaneously, and with the same speed of transition from $\mu_{1,s}$ and $\phi_{1,i}$ to $\mu_{2,s}$ and $\phi_{2,i}$ respectively, $s = 1, \ldots, S$ and $i = 1, \ldots, p$.

3 Specification of Seasonal FISTAR Model

For non-linear time series models, Granger (1993) has suggested the use of a "specific to general" procedure. To this aim, we extend the specification procedure elaborated by Teräsvirta (1994) for STAR models and van Dijk et al. (2002) for FISTAR models to specify the empirical representation of the seasonal FISTAR model in (2.1)–(2.2).

For a given data, our empirical specification consists of the following steps: First, we specify the autoregressive order p for the adequate seasonal ARFI model using the BIC criterion. Second, we test the null hypothesis of linearity against seasonal

FISTAR model and select the appropriate transition function. Next, we estimate the seasonal FISTAR model. Finally, we evaluate the model using diagnostic tests.

3.1 Nonlinearity Test

The null hypothesis of linearity holds if the seasonal and the autoregressive parameters are constant over the different regimes, i.e., $H_0 : \mu_{1,s} = \mu_{2,s}$ and $\phi_{1,i} = \phi_{2,i}$, $i = 1, \dots, p$ and $s = 1, \dots, S$. This null hypothesis can be expressed in another way: $H_0' : \gamma_F = \gamma_G = 0$. The alternative hypothesis of non-linearity is given by $H_1 : \mu_{1,s} \neq \mu_{2,s}$ and/or $\phi_{1,i} \neq \phi_{2,i}$, for at least one value of i, s.

It is clear from (2.1)–(2.2), that under the null hypothesis H_0', the parameters γ_F, c_F, γ_G and c_G are not identified. To resolve this problem, Luukkonen et al. (1988) propose to replace the transition functions in (2.2) by their first order Taylor expansion with respect to γ around 0. However, they have showed that LM_1 test is enable to detect nonlinearity when only the intercept differs across regimes. To overcome this problem, they have suggested to approximate the transition functions by their third order Taylor approximation. This yields the auxiliary regression:

$$\begin{aligned} x_t = {} & \beta_0' D_t + \beta_1' D_t S_{F,t} + \beta_2' D_t S_{F,t}^2 + \beta_3' D_t S_{F,t}^3 + \delta_0' w_t \\ & + \delta_1' w_t S_{G,t} + \delta_2' w_t S_{G,t}^2 + \delta_3' w_t S_{G,t}^3 + e_t \end{aligned} \tag{3.1}$$

where $D_t = (D_{1,t}, \dots, D_{S,t})'$, $w_t = (x_{t-1}, \dots, x_{t-p})'$, $\beta_i = (\beta_{i,1}, \dots, \beta_{i,S})'$, $\delta_i = (\delta_{i,1}, \dots, \delta_{i,p})'$, $i = 0, 1, 2, 3$ and e_t is the residual terms such that under H_0, $e_t = \varepsilon_t$. Hence, the null hypothesis $H_0{}'$ is equivalent to test $H_0{}'' : \beta_1 = \beta_2 = \beta_2 = \delta_1 = \delta_2 = \delta_3 = 0$. This null hypothesis of linearity can be tested using Lagrange Multiplier $[LM_3]$ statistic. The conditional log-likelihood, by assuming the normal distribution of the errors, for observation t is written as:

$$l_t = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma^2) - \frac{e_t^2}{2\sigma^2} \tag{3.2}$$

Under the linearity hypothesis H_0, the remaining partial derivatives are given by: $\left.\frac{\partial l_t}{\partial \beta_i}\right|_{H_0} = \frac{1}{\sigma^2}\hat{\varepsilon}_t D_t S_{F,t}^i$, $\left.\frac{\partial l_t}{\partial \delta_i}\right|_{H_0} = \frac{1}{\sigma^2}\hat{\varepsilon}_t w_t S_{G,t}^i$ and $\left.\frac{\partial l_t}{\partial d}\right|_{H_0} = -\frac{1}{\sigma^2}\hat{\varepsilon}_t \sum_{j=1}^{t-1} \frac{\hat{\varepsilon}_{t-j}}{j}$ with $i = 0, 1, 2, 3$. Where $\hat{\varepsilon}_t$ are the residuals obtained from the seasonal ARFI model.

To constract the LM_3 test for nonlinearity we follow Teräsvirta (1994) and van Dijk et al. (2002). We estimate the seasonal ARFI model and we compute the residuals $\hat{\varepsilon}_t$ and the sum of squared residuals $SSR_0 = \sum_{j=1}^{T} \hat{\varepsilon}_t^2$. Next, we regress $\hat{\varepsilon}_t$ on $D_t S_{F,t}^i$, $w_t S_{G,t}^i$ and $-\sum_{j=1}^{t-1} \frac{\hat{\varepsilon}_{t-j}}{j}$, $i = 0, 1, 2, 3$; and we compute the sum of squared residuals from this regression SSR_1. The LM_3 statistic is then given by $LM_3 = \frac{SSR_0 - SSR_1}{SSR_1/T} \rightsquigarrow \chi^2(3(p+S))$.

For (2.6) and (2.7), auxiliary regressions and nonlinearity LM test can be performed as for the general model. The nonlinearity test, discussed above, is carried out for different transition variables. When the null hypothesis is rejected, we select the most appropriate transition variable based on the LM statistic values. That is, we select the one with the smallest p-value.

3.2 Estimation

In this section we discuss the estimation method for the seasonal FISTAR model. When the transition variable is selected from the non-linearity test in Sect. 2, we can estimate the seasonal FISTAR model using Beran's (1995) approximate maximum likelihood estimator for ARFIMA model which was adapted by van Dijk et al. (2002) to estimate the FISTAR model. The estimated parameters $\hat{\theta}$ are obtained by minimizing the sum of squared residuals:

$$Q_T(\theta) = \sum_{t=1}^{T} e_t^2(\theta) \tag{3.3}$$

where $\theta = (\mu_1', \mu_2', \phi_1', \phi_2', \gamma_F, \gamma_G, c_F, c_G, d)'$, $\mu_j = (\mu_{j,1}, \ldots, \mu_{j,S})'$, $\phi_j = (\phi_{j,1}, \ldots, \phi_{j,p})'$, $j = 1, 2$, and $e_t(\theta)$ are the residuals from the seasonal FISTAR model.

Notice that the seasonal parameters μ_1 and μ_2 and the autoregressive parameters ϕ_1 and ϕ_2 can be estimated, conditional upon fractional parameter d and the parameters of transition functions γ_F, γ_G, c_F and c_G, using Ordinary Least Square estimator (see van Dijk et al. 2002).

Starting values needed in the optimization algorithm can be obtained using five dimensional grid search over d, γ_F, γ_G, c_F and c_G. The selected starting values are those that give the smallest estimator for the residuals variance $\hat{\sigma}^2(\gamma_F, \gamma_G, c_F, c_G, d)$.

For the cases when $\gamma_F = 0$ or when $F(S_{F,t}; \gamma_F, c_F) = G(S_{G,t}; \gamma_G, c_G)$, the estimation procedure is performed in a straightforward manner.

3.3 Misspecification Test

To evaluate the fitted seasonal FISTAR model, it is required to make some diagnostic tests on the resulting residuals. Specially by testing the residuals serial correlation. In this section, we extend the LM test for STAR model residuals correlation proposed by Eitrheim and Teräsvirta (1996).

The null hypothesis of no residual autocorrelation for seasonal FISTAR model can be tested against the alternative of serial dependence up to order q, given by

$$\varepsilon_t = \left(\sum_{j=1}^{q} a_j L^j \right) \varepsilon_t + v_t, \tag{3.4}$$

with ε_t are the residuals from the seasonal FISTAR model, $v_t \sim iid\left(0, \sigma^2\right)$ and L is the lag operator. The hypothesis of serial independence of ε_t, $H_0 : a_1 = a_2 = \ldots =$

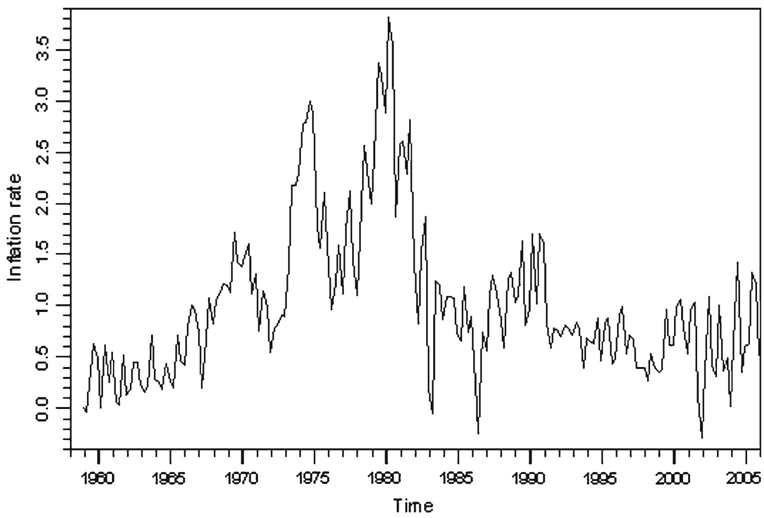

Fig. 1 Quarterly seasonally unadjusted US inflation rate

$a_q = 0$ can be tested by an LM test (see Eitrheim and Teräsvirta 1996). We modify the no residual correlation LM test for STAR model by including the gradients of v_t with respect to the seasonal parameters, the parameters of transition function $F\left(S_{F,t}; \gamma_F, c_F\right)$ and the fractional differencing parameter d.

4 Empirical Application

4.1 Data

Our data consists of quarterly USA unadjusted consumer price index based inflation rates from 1958–3 to 2005–4. These data are obtained from the IMF's International Financial Statistics. Inflation rates are constructed by taking 100 time the first difference of the natural log of the consumer price index. The inflation series with 189 observations is depicted in Fig. 1 and looking for this plot, it is clear that inflation exhibits a nonlinearity behavior.

4.2 Empirical Specification

4.2.1 Nonlinearity Test

In this section, we start by selecting the autoregressive order for a seasonal linear $ARFI$ model to the US inflation rates. This model is estimated for different autoregressive orders. The appropriate one is given by the BIC criterion. We obtain $p = 3$ as the adequate for a maximum autoregressive order $p_{max} = 5$.

The next step consists in testing linearity against seasonal $FISTAR$, based on LM_k-type tests, where $k = 1$ and 3 is the order Taylor approximation. Linearity is tested

Table 1 LM-type tests of nonlinearity

Transition variable	LM_1	LM_3
Seasonal $FISTAR$ ($F = \frac{1}{2}$)		
$\triangle_4 y_{t-1}$	0.052	0.036
$\triangle_4 y_{t-2}$	0.191	0.072
$\triangle_4 y_{t-3}$	0.010	0.009
$\triangle_4 y_{t-4}$	0.772	0.184
Seasonal $FISTAR$ ($F = G$)		
$\triangle_4 y_{t-1}$	0.029	8×10^{-4}
$\triangle_4 y_{t-2}$	0.046	0.023
$\triangle_4 y_{t-3}$	0.036	0.031
$\triangle_4 y_{t-4}$	0.638	0.127
Seasonal $FISTAR$ ($F \neq G$)		
$\triangle_4 y_{t-2}$ $\triangle_4 y_{t-3}$	0.007	–
$\triangle_4 y_{t-1}$ $\triangle_4 y_{t-1}$	–	7×10^{-4}

Note. The table contains p-values of LM-type statistics nonlinearity where LM_k, $k = 1$ and 3, denote the LM-type test for nonlinearity based on an k-th order Taylor approximation of F and G. For seasonal FISTAR ($F \neq G$) we have computed LM_k, $k = 1$ and 3 for 16 pairs ($\Delta_4 y_{t-i}$, $\Delta_4 y_{t-j}$) where the first element of each pairs is the transition variable of F and the second is the one of G with $1 \leq i, j \leq 4$ and we give the choice which corresponds to the minimal p-value

against the three representations (2.6), (2.7) and (2.2). Transition variables should be free from seasonality (see Franses et al. 2000). Thus we use as potential transition variables seasonal difference of y_t, that is, $\triangle_S y_{t-l} = y_{t-l} - y_{t-l-S}$ with $l = 1, \ldots, 4$.

Table 1 contains p-values of LM_k-type tests for different representations of the seasonal $FISTAR$ models (2.6), (2.7) and (2.2). For the first representation, i.e. when $F = \frac{1}{2}$, as in Eq. 2.6, the null hypothesis of linearity is tested by LM_1 and LM_3 tests for the different transition variables. Linearity is rejected at 5% level for both transition variables $\triangle_4 y_{t-1}$ and $\triangle_4 y_{t-3}$ where the minimum p-values are obtained with $\triangle_4 y_{t-3}$.

For the second representation (2.7), i.e. $F = G$, the null hypothesis of linearity is rejected at 5% significance level, based on both LM_1 and LM_3 tests, for transition variables $S_t = \triangle_4 y_{t-l}$, for $l = 1, 2, 3$. Based on LM p-value, we select $\triangle_4 y_{t-1}$ as transition variable.

For general model (2.2), the results of LM-type test for all possibility of transition variable pairs $S_{F,t} = \triangle_4 y_{t-d_F}$ and $S_{G,t} = \triangle_4 y_{t-d_G}$, $d_F = 1, \ldots, 4$ and $d_G = 1, \ldots, 4$, indicate that the appropriate transition variable is $\triangle_4 y_{t-1}$ for both transition functions F and G.

4.2.2 Estimation

When the transition variables are given from LM-type tests, seasonal $FISTAR$ model is estimated for (2.6), (2.7) and (2.2) using approximate maximum likelihood method discussed in Sect. 3. We also give the estimation results of a linear seasonal $ARFI$ model to make comparison with nonlinear models. We will select the appropriate model between them to describe US inflation rates behaviour.

The estimation results of different models are reported in Table 2. In the second column of Table 2, we present the results of seasonal $ARFI$ estimation. The estimated

Table 2 Summary of estimated models for US inflation rates

Parameters	$SEA\text{-}ARFI$		$SEA\text{-}FISTAR$ $(F = \frac{1}{2})$		$SEA\text{-}FISTAR$ $(F = G)$		$SEA\text{-}FISTAR$ $(F \neq G)$	
$\hat{d}$	0.21	(0.189)	0.25	(0.046)	0.38	(0.092)	0.29	(0.162)
$\hat{c}_G$	–	–	−0.46	(0.001)	0.61	(0.007)	0.39	(0.131)
$\hat{\gamma}_G$	–	–	1,397.60	(–)	127.20	(208.141)	33.64	(252.8)
$\hat{c}_F$	–	–	–	–	–	–	−0.15	(0.043)
$\hat{\gamma}_F$	–	–	–	–	–	–	52.17	(181.7)
$\hat{\phi}_{1,1}$	0.50	(0.177)	0.278	(0.161)	0.257	(0.120)	0.394	(0.189)
$\hat{\phi}_{1,2}$	−0.12	(0.077)	−0.393	(0.134)	−0.219	(0.077)	−0.161	(0.082)
$\hat{\phi}_{1,3}$	0.41	(0.068)	0.054	(0.146)	0.235	(0.084)	0.288	(0.091)
$\hat{\phi}_{2,1}$	–	–	0.468	(0.083)	0.033	(0.279)	0.283	(0.162)
$\hat{\phi}_{2,2}$	–	–	−0.041	(0.086)	0.294	(0.215)	0.108	(0.163)
$\hat{\phi}_{2,3}$	–	–	0.404	(0.082)	1.093	(0.215)	0.554	(0.132)
$\hat{\mu}_{1,1}$	0.072	(0.069)	−0.079	(0.065)	−0.061	(0.065)	−0.031	(0.120)
$\hat{\mu}_{1,2}$	0.076	(0.069)	0.060	(0.068)	0.047	(0.065)	0.135	(0.121)
$\hat{\mu}_{1,3}$	0.241	(0.063)	0.241	(0.060)	0.247	(0.068)	0.221	(0.107)
$\hat{\mu}_{1,4}$	0.086	(0.066)	0.047	(0.063)	0.069	(0.066)	0.342	(0.107)
$\hat{\mu}_{2,1}$	–	–	–	–	−0.715	(0.217)	−0.133	(0.082)
$\hat{\mu}_{2,2}$	–	–	–	–	−0.508	(0.231)	0.001	(0.081)
$\hat{\mu}_{2,3}$	–	–	–	–	0.048	(0.198)	0.261	(0.082)
$\hat{\mu}_{2,4}$	–	–	–	–	0.141	(0.307)	−0.052	(0.094)

Note. Standard errors are given in parentheses

fractional integration parameter $\hat{d}$ is equal to 0.21 but is not significant at 5% level. The seasonal changing in the estimated means confirms the existence of seasonal behaviour in the inflation rates.

The estimation results of the seasonal $FISTAR$ model specifications in (2.6), (2.7) and (2.2) are given in column 3, 4 and 5 of Table 2 respectively. The estimated differencing parameter $\hat{d}$ is equal to 0.25 for (2.6), 0.38 for (2.7) and 0.29 for (2.2). They are significant at 5% level except for $\hat{d}$ in (2.2) which is significant at 10% level. This suggests strong evidence of long memory in inflation rates. The estimated threshold parameter $\hat{c}$ in (2.6) and (2.7) are equal to −0.46 and 0.61 respectively. For (2.2) the threshold parameters $\hat{c}_G$ and $\hat{c}_F$ are equal to 0.39 and −0.15 respectively. All threshold parameters are significant at 5% level. Comparing $\hat{\phi}_{1,i}$ with $\hat{\phi}_{2,i}$ and $\hat{\mu}_{1,s}$ with $\hat{\mu}_{2,s}$, we can observe that in all cases there are different regimes in both seasonal and autoregressive parameters.

4.3 Diagnostic and Comparison

The diagnostic on the different estimated models is based on the properties of resulting residuals. Three different tests are used to this aim: Jarque Bera normality test, residuals autocorrelation test as described in Sect. 2 and finally a test for $ARCH$ effect.

Table 3 presents the different diagnostic results for the different models. Skewness, Kurtosis and Jarque Bera statistics show that we cannot reject the hypothesis of normality at 5% except for (2.6) and (2.2). Residuals autocorrelation test based on LM statistics for seasonal $FISTAR$ models and based on Ljung-Box statistics for

Table 3 Misspecification tests for estimated models

	$SEA\text{-}ARFI$		$SEA\text{-}FISTAR$ $(F=\frac{1}{2})$		$SEA\text{-}FISTAR$ $(F=G)$		$SEA\text{-}FISTAR$ $(F\neq G)$	
SSR	26.238		23.559		21.842		22.207	
AIC	−1.831		−1.88		−1.913		−1.875	
BIC	−1.688		−1.65		−1.610		−1.536	
SK	−0.308		−0.475		−0.216		−0.581	
Kur	3.405		3.759		3.487		4.014	
JB	4.06	(0.131)	11.03	(0.04)	3.17	(0.205)	17.75	(0.0001)
$ARCH(1)$	3.28	(0.07)	1.092	(0.296)	8.44	(0.0037)	3.06	(0.08)
$ARCH(4)$	12.56	(0.014)	1.912	(0.752)	15.31	(0.004)	6.34	(0.174)
$LM_{SC}(1)$	0.014	(0.906)	0.91	(0.34)	3.03	(0.08)	8.95	(0.003)
$LM_{SC}(2)$	0.061	(0.970)	1.46	(0.24)	2.39	(0.09)	7.28	(0.001)
$LM_{SC}(3)$	0.768	(0.857)	1.03	(0.38)	2.09	(0.10)	4.8	(0.003)
$LM_{SC}(4)$	0.974	(0.914)	0.84	(0.50)	1.56	(0.19)	3.76	(0.006)
$LM_{SC}(8)$	7.418	(0.492)	0.99	(0.45)	1.10	(0.37)	2.62	(0.010)

Note. The table presents misspecification tests for estimated models. SSR denotes the sum of squared residuals, SK is skewness, Kur is Kurtosis, JB the Jarque-Bera test of normality of the residuals, $ARCH(q)$ is the LM test of no autoregressive conditional heteroscedasticity up to order q, and $LM_{SC}(q)$ denotes the Ljung-Box statistics and F variant of LM test of no serial correlation in residuals up to order q for Seasonal ARFI and Seasonal FISTAR, respectively. The numbers in parentheses represent p-values

seasonal $ARFI$ model provide strong evidence for no residual autocorrelation except for representation (2.2). The test of residual $ARCH$ effect does not reject the homoskedasticity hypothesis any longer for $ARFI$ model and seasonal $FISTAR$ model (2.7). Finally, results of residuals diagnostics suggest that BIC prefers $ARFI$ model. Whereas seasonal $FISTAR$ model (2.7) has the smallest AIC and sum of squared residuals. In addition, the long memory parameter estimator $\hat{d}$ in $ARFI$ model is not significantly different from 0. Thus we can conclude that the seasonal $FISTAR$ model (2.7) is the most appropriate one to describe the US inflation rates behaviour.

In Fig. 2, we present the transition function in the selected FISTAR $(F = G)$ over transition variable $\Delta_4 y_{t-1}$. As suggested by a large value of the estimated parameter $\hat{\gamma}$, transition from one regime to another occurs suddenly at the estimated threshold $\hat{c}$. The transition function over time is presented in Fig. 3. This transition function can give further insight into the cyclical behaviour of inflation rates. G_t gives the probability of the upper regime. When the probabilities are greater than 0.5, inflation could be judged to be in the upper regime. It seems that transition function has similar dynamics to the large value of inflation annual growth. However, the change of transition function does not coincide with that of inflation because we use seasonal differenced inflation as transition variable.

5 Conclusion

In this paper, we have defined a Seasonal Fractional Integrated Smooth Transition model (SEA-FISTAR model) that allows for regime switching, long memory and deterministic seasonality behaviours. We have used the specific to general procedure to select the appropriate model for US quarterly inflation rates. We conclude that linear

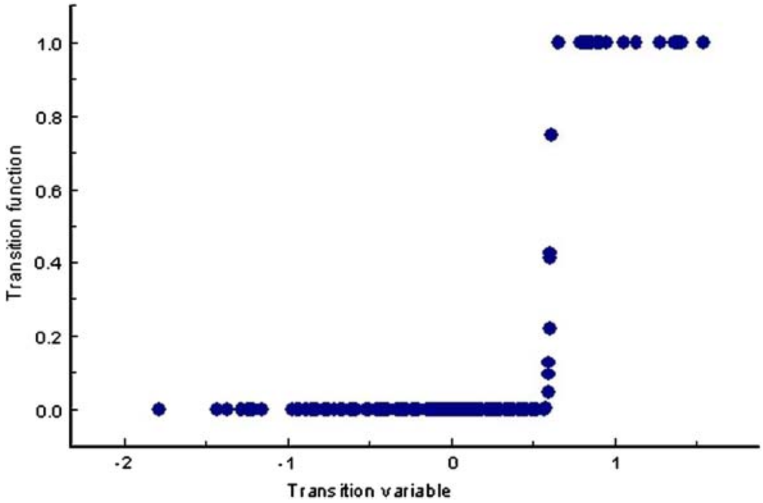

Fig. 2 Transition function versus $\triangle_4 y_{t-1}$

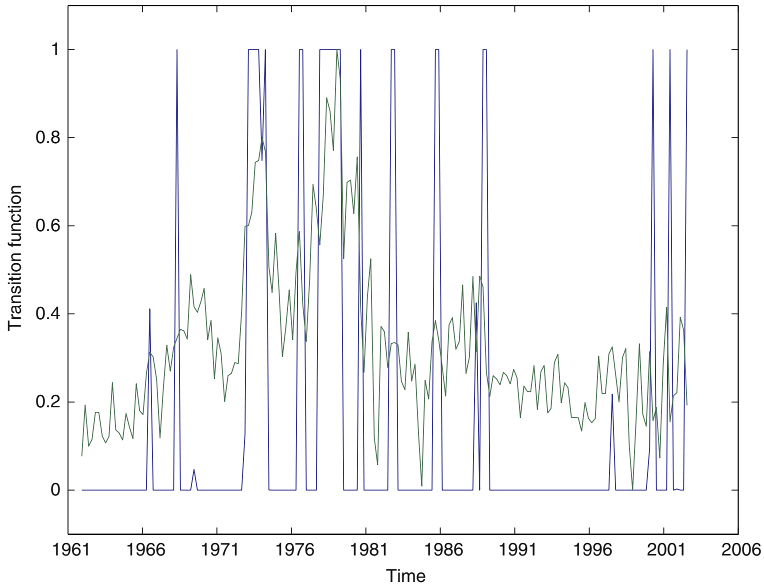

Fig. 3 Transition function versus time

seasonal ARFI model is outperformed by the nonlinear seasonal FISTAR model for describing US inflation rates. An important feature we point out is that the seasonal component is time dependent which changes with regimes; such behaviour can not be identified if we consider seasonally adjusted data.

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